

Chaos and High-Energy Collisions

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The investigations of the gauge boson wave packets collisions in SU(2) Higgs model are presented. The evolution of the gauge and Higgs fields is studied as a function of the amplitude and the mass ratio of M_H/M_W . We visualize the restoration of spontaneously broken gauge symmetry during collisions.

1 Introduction. Infrared Instabilities and Dynamical Chaos

Recently, the nonperturbative treatment of the high multiplicity scattering amplitudes which are not suppressed in the weak coupling limit has attracted attention^{1,2,3,4}. It is quite natural in the connection with the exciting expectations that the rate of the baryon-number violating electroweak processes, non-perturbative in essence⁵, might be significant at ultra-high energies⁶. The semiclassical technique, however, meets with difficulties in the study of $2 \rightarrow$ many particles amplitude, since the *initial* state of few highly energetic particles is not semiclassical at all (see, e.g.⁷).

In the extreme non-perturbative classical treatment of the high energy multiparticle amplitude, the question is the following: Does there exist a mechanism for energy transfer from high frequency modes, corresponding to two (or few) initial high energy particles, to low frequency modes representing a multiparticle final states? At first glance, the answer to this question, formulated in terms of nonlinear dynamics, seems to be affirmative since the gauge field equations are nonlinear. However, the studies of $(1+1)$ -dimensional Abelian Higgs model¹ and $\lambda\phi^4$ -theory² have shown no indication for a mechanism providing the coupling between the initial high and the final low frequency modes.

Of course, the gauge field nonlinearities inherent in the non-Abelian gauge theories and which are absent in the Abelian models,^a are essential and, in general, lead to the infrared instabilities.

One may say that for non-Abelian gauge theories the infrared instabilities are not an exception, but rather a rule, and they are intimately connected with i) the masslessness of the gauge field, ii) its isospin (color) charge, and iii) the

^aThe nonlinearities of Abelian models due to the Higgs-gauge fields and the Higgs self-coupling, as we see below, are not important at the high energy gauge boson collisions.

gyromagnetic ratio of the gluon equal 2. Nonlinearity itself is not enough to furnish the above mentioned coupling between fast and slow modes, as one can argue from negative results of ^{1,2}.

From a more general point of view, the observed inability of the nonlinearity alone to provide a mechanism for the formation of the inelastic final states is intimately connected with the integrable nature of the classical systems considered in ^{1,2}. It is well known that non-Abelian gauge theories are nonintegrable in the classical limit and exhibit dynamical chaos ^{8,9} (see also ¹⁰ for details and extended literature).

This dynamical stochasticity of the non-Abelian gauge fields together with their mentioned instability are two possible sources of the mechanism for the coupling between high and low frequency modes.^b At the same time, it is not superfluous to recall the role of the Higgs condensate in the suppressing of the chaos of the non-Abelian gauge fields ¹¹.

In ⁴ we studied the collision of two SU(2) gauge field wave packets. As we expected, based on our previous results ³, the collisions of essentially non-Abelian initial configurations trigger the decay of initial states into many low frequency modes with dramatically different momentum distributions, whereas for Abelian configurations wave packets pass through each other without interaction.

Here I will present the study of the collisions of wave packets in the SU(2) Higgs model where the fundamental excitations of the gauge-field are massive.

2 Collisions of Classical Wave Packets in SU(2) Higgs Model

2.1 SU(2) Higgs Model

We briefly describe the spontaneously broken SU(2) model with an isodoublet Higgs field Φ . This model retains the most relevant ingredients of the electroweak theory. The action of this model is given by

$$S = \int d^3x dt \left\{ -\frac{1}{2} \text{tr} (F_{\mu\nu} F^{\mu\nu}) + \frac{1}{2} \text{tr} [(\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi] - \lambda \left[\frac{1}{2} \text{tr} (\Phi^\dagger \Phi) - v^2 \right]^2 \right\}. \quad (1)$$

with $\mathcal{D}_\mu = \partial_\mu - ig A_\mu^a \tau^a / 2$, $F_{\mu\nu} \equiv F_{\mu\nu}^a \frac{\tau^a}{2} = \frac{i}{g} [\mathcal{D}_\mu, \mathcal{D}_\nu]$ and $\Phi = \phi^0 - i\tau^a \phi^a$; τ^a ($a = 1, 2, 3$) are Pauli matrices. v is a vacuum expectation value (*v.e.v.*) of

^bIt is not excluded that these two sources have a common deep origin, though I am unable to prove this assertion on theorem-like grounds.

the neutral component of the scalar field.

By proper scaling transformations of the space-time coordinates and fields, it is easy to see that the action (1) and the corresponding equations of motion possesses only a single parameter

$$\frac{\lambda}{g^2} = \frac{M_H^2}{8M_W^2} \quad (2)$$

($M_H = 2v\sqrt{\lambda}$ and $M_W = \frac{gv}{\sqrt{2}}$ are the tree masses of Higgs and gauge W -bosons respectively; $v = 174$ GeV.) However, in the simulation of the wave packet collisions, initial conditions introduce extra physical parameters.

We work in the unitary gauge where only physical excitations appear:

$$\begin{aligned} \Phi &= \left(v + \frac{\rho}{\sqrt{2}} \right) U(\theta) \\ A_\mu &= U(\theta) W_\mu U^{-1}(\theta) + \frac{i}{g} (\partial_\mu U(\theta)) U^{-1}(\theta) \end{aligned} \quad (3)$$

with $U(\theta) = \exp(i\tau^a \theta^a)$.

The real field ρ describes the oscillations of the scalar field about its *v.e.v.*, and W_μ is the W -boson field:

$$(\partial_\mu \partial^\mu + M_H^2) \rho + 3\sqrt{2}\lambda v \rho^2 + \lambda \rho^3 - \frac{1}{4} g^2 W_\mu^a W^{a\mu} \rho^2 - \frac{1}{2\sqrt{2}} g^2 v W_\mu^a W^{a\mu} = 0, \quad (4)$$

$$[\mathcal{D}_\mu, F^{\mu\nu}] + \left(M_W^2 + \frac{1}{\sqrt{2}} g^2 v \rho + \frac{1}{4} g^2 \rho^2 \right) W^\nu = 0. \quad (5)$$

We emphasize that the gauge field acts as a source for Higgs excitations in (4) (last term in (4)). This permits us to consider the W -field classically.

2.2 Scattering of Wave Packets

Our numerical study is based on the Hamiltonian formulation of lattice SU(2) gauge theory¹³ (see^{4,14} for details). We work on an one-dimensional lattice with a size $L = Na$ (N is the number of lattice sites, a is the lattice spacing).

To implement the temporal gauge $W_0^a = 0$, most convenient in the Hamiltonian formulation of the lattice gauge theory, we “collide” transverse W -bosons, for which the relation $\partial_\mu W^{a\mu} = 0$ holds.

The initial configuration is given by two well-separated right- and left-moving Gaussian wave packets originally centered at $z_{R(L)}$ with average momenta $\mathbf{k} = (0, 0, \bar{k})$ and width Δk ($\Delta k \ll \bar{k}$):

$$W^{c,\mu} = W_R^{c,\mu} + W_L^{c,\mu}$$

$$W_{R(L)}^{c,\mu} = \delta^{\mu 2} n_{R(L)}^c \psi(z - z_{R(L)}, \pm t) \quad (6)$$

with $n_{R(L)}^c$ the unit isospin vectors. To specify the profile function $\psi(z, t)$ we take, for a right- moving wave packet centered at $z = 0$ at $t = 0$,

$$\psi(z, t) = \frac{1}{\sqrt{\pi^{3/2} \bar{\omega} \Delta k \sigma}} \int_{-\infty}^{\infty} dk e^{-(k-\bar{k})^2/2(\Delta k)^2} \cos(\omega t - kz) \quad (7)$$

with $\omega = (k^2 + M_W^2)^{1/2}$, and the normalization is fixed by requiring energy equal to $\bar{\omega}$ per cross-sectional area σ .

From (7) we get the initial conditions for $\psi(z, 0)$ and $\frac{d\psi}{dt}(z, t)|_{t=0}$ which we don't give explicitly here. The initial condition for the Higgs field is given by the vacuum solution at $t = 0$: $\phi^0 = v$, $\phi^a = 0$, $\dot{\phi}^0 = \dot{\phi}^a = 0$. ($\cdot \equiv \frac{d}{dt}$). It is possible to see that the initial conditions introduce three new dimensionless parameters \bar{k}/v , $\frac{\Delta k}{v}$ and $\frac{\sigma v^2}{g^2}$ in addition to $\frac{\lambda}{g^2}$ (or M_H/M_W) (we fix in the following $g = 0.65$).

One more parameter appears in the initial conditions—the angle θ_c between the relative orientation of isospins of two wave packets. Non-zero θ_c corresponds to the essentially non-Abelian configuration, $\theta_c = 0$ (parallel isospins) gives the initial pure Abelian configurations.

For the pure Yang-Mills wave packet collisions, the non-linearity is due to the self-interaction of gauge fields. As it was established in⁴ for $\theta_c = 0$ no indications of the final inelastic states had a place: wave packets passed through each other without interaction. On the contrary, for $\theta_c \neq 0$ collisions resulted in strongly inelastic final states⁴. It is remarkable that the inelastic patterns remain qualitatively similar for θ_c as small as $\sim 10^{-12}$, clearly connecting these phenomena with the dynamical chaos of the non-Abelian gauge fields.

For the Yang-Mills-Higgs system, the situation is more involved due to the additional non-linearities induced by the gauge field-Higgs and the Higgs self-couplings.

Figures 1 and 2 (top rows) show a few “snapshots” of the space-time development of the colliding W -boson wave packets for parallel (Fig. 1) and orthogonal (Fig. 2) isospin orientations. The figures show the absolute value of the scaled gauge field amplitude $|A|/v$. For parallel isospin orientations, the result of the “collisions” is a slight distortion of the initial wave packets showing no sign of the inelasticity. Decreasing of the energy ($\bar{k} = \pi/25$) shows a small inelasticity for $\theta_c = 0$.^c As is seen from the top row of Fig. 2, for $\theta_c = \pi/2$, final states are strongly inelastic.

^cThis fact is easily explained by the consideration of the tree diagrams in WW -scattering since the scalar exchanges are decreased with energy.

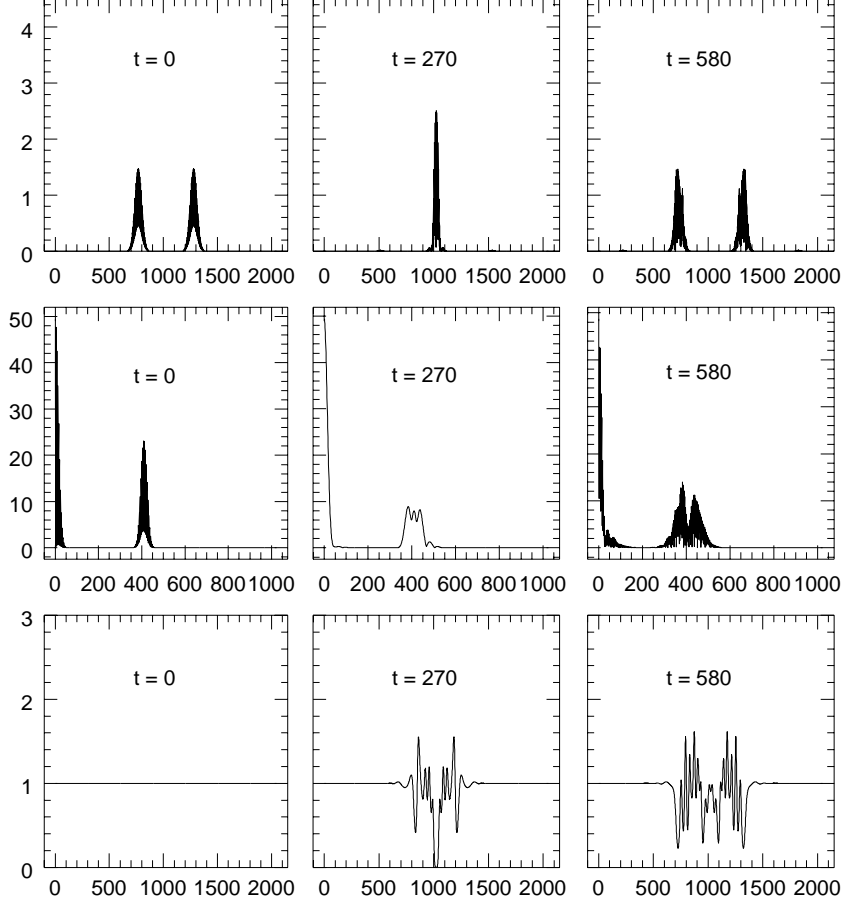


Figure 1: Collision of two W -wave packets with parallel isospin polarizations. We choose $M_H = M_W = 0.126$, $\bar{k} = \pi/5$, $\Delta k = \pi/100$, $g = 0.65$, and $\sigma = 0.336$. This simulation, as well as all others below, was performed on a lattice of length $L = 2048$ and lattice spacing $a = 1$. The top row shows the space-time evolution of the scaled gauge field amplitude $|A|/v$, the median row exhibits the corresponding Fourier spectra of the gauge field energy density, and the bottom row shows the space-time evolution of the scaled Higgs field $|\Phi|^2/v^2$. The abscissae of top and bottom rows are labelled in units of the lattice spacing, and the abscissa of the median row is in units of $\sigma/1024$.

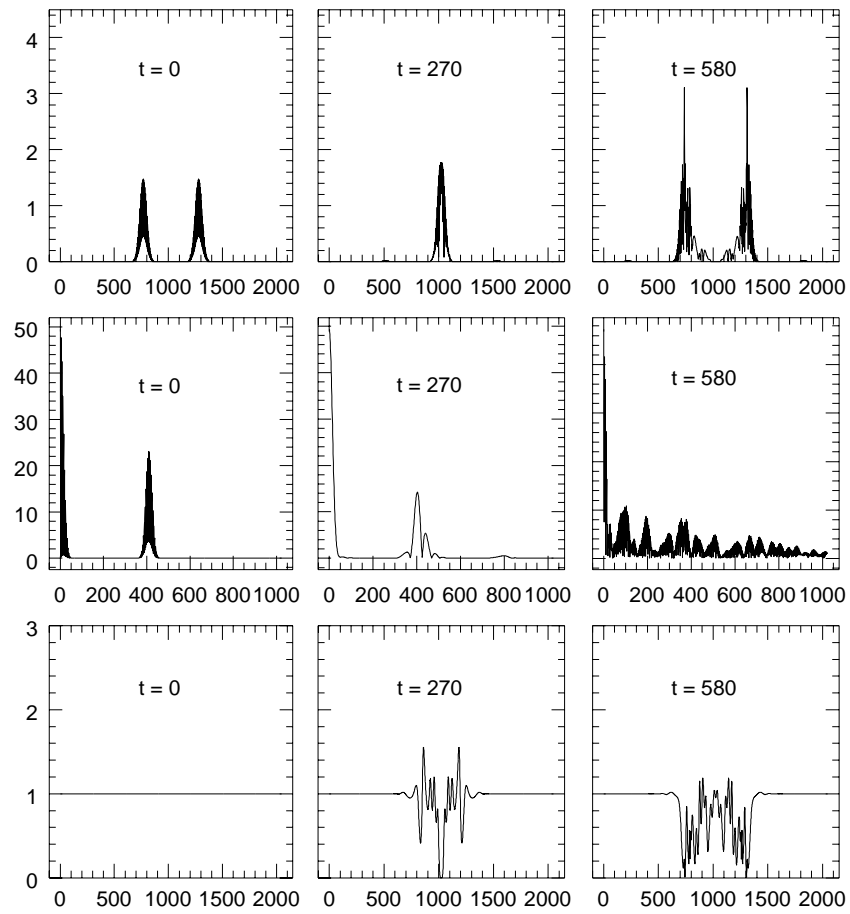


Figure 2: Same as for Figure 1, for orthogonal polarizations.

The difference between the two cases ($\theta_c = 0, \pi/2$) is more striking by looking at the evolution of the absolute value of the Fourier transform of the gauge-invariant energy density (scaled by v^2) (median row in Figs. 1 and 2).

The bottom rows of Figs. 1 and 2 illustrate the time evolution of the Higgs field excitations around its *v.e.v.* v (scaled to unity). Here we have plotted the $|\Phi|^2/v^2$ as a function of space coordinate at three different times.

3 Symmetry Restoration in WW collisions

As is seen from Figures 1-2 (bottom rows), for not very large $r = M_H/M_W$ the Higgs field oscillates not about its *v.e.v.* v but rather about zero. This suggests that the collisions of the gauge boson wave packets, accompanied by energy transfer from gauge field to Higgs field, lead to the restoration of the broken SU(2) symmetry. This phenomenon occurs for the large gauge fields amplitude (see ^{16,17}). Indeed, (4), describing the excitations of the scalar field about the Higgs vacuum $|\Phi| = v$, has another exact solution $\rho = -\sqrt{2}v$ ($|\Phi| = 0$, see (3)) with W_μ^c being arbitrary. In terms of the small excitations $\chi = \rho + v/\sqrt{2}$ about $\Phi = 0$, for (4) and (5) we have:

$$\left[\partial_\mu \partial^\mu - \frac{M_H^2}{2} \left(1 + \frac{g^2 W^2}{8\lambda v^2} \right) \right] \chi + \lambda \chi^3 = 0 \quad (8)$$

$$[\mathcal{D}_\mu, F^{\mu\nu}] + \frac{1}{4} g^2 \chi^2 W^\nu = 0 \quad (9)$$

where $W^2(x) \equiv W_\mu^a(x) W^{a\mu}(x) = -(W_i^a(x))^2 < 0$ for our choice of the transverse W -wave packets. $W^2(x)$ is always negative for time-like bosons.^d

Eq. (9) describes the massless W -boson. From (8) and (9), we have the effective potential for χ -excitations:

$$V(\chi, W^2) = -\lambda v^2(1 - \eta)\chi^2 + \frac{\lambda}{4}\chi^4, \quad (10)$$

where we introduce $\eta = \frac{g(W_i^a)^2}{8\lambda v^2} = \frac{1}{r^2} \langle W_i^{a^2} \rangle$ as a parameter where the intensity $(W_i^a)^2$ of the high frequency gauge pulses is replaced by its space-time average $\langle W^2 \rangle$. Depending on whether $\eta > 1$ or $\eta < 1$, the potential (10) has two different *stable* minima:

$$\eta < 1: \quad \chi_{\min} = \pm \sqrt{2}v(1 - \eta)^{1/2}, \quad \text{i.e. } |\phi| = v(1 - \eta)^{1/2}, \quad (11)$$

$$\eta > 1: \quad \chi_{\min} = 0, \quad \text{i.e. } \Phi = 0. \quad (12)$$

^dWe recall that the luminosity of transverse W -bosons generated by the energetic fermions is much higher than that of longitudinal ones and increases with energy ¹⁸.

Stable excitations about these “vacua” have the following squared masses:

$$\tilde{M}_W^2 = M_W^2(1 - \eta)\theta(1 - \eta) \quad (13)$$

$$\tilde{M}_W^2 = \frac{M_H^2}{2}|1 - \eta|[1 + \theta(1 - \eta)]. \quad (14)$$

Thus for $\eta > 1$, the symmetry is restored and the scalar oscillations occur about the symmetrical state $\Phi = 0$ with the zero effective mass of the W -boson.

For $\eta < 1$, the vacuum is changed gradually as $(1 - \eta)^{1/2}$. For this case, $\tilde{r} = \tilde{M}_H/\tilde{M}_W = r$. In Figure 3, first column, the space development of the WW collision is shown for $\eta = 1.32$ for different times. As seen from this figure, at time $t \approx 300$ wave packets collide and then begin to separate. Just about at this time one expects to observe the restoration of symmetry, i.e. the oscillations of the scalar field ϕ about a new ground state located below the “old” vacuum $|\phi| = v$. After the separation of the wave packets ($t > 300$) the scalar field excitations tend again to oscillate about “old” vacuum, i.e. the gauge symmetry is broken again. The second column exhibits the space-time evolution of the $|\phi|/v$. The third column shows the Higgs field smoothed over 50 lattice sites in order to facilitate a comparison with the definition (10) of the parameter η in terms of the averaged strength of the W -boson field.

4 Concluding Remarks

In the numerical studies of the collisions classical wave packets of transversely polarized gauge bosons with non-parallel isospin orientations in the broken $SU(2)$ gauge theory we have found evidence for the creation of final states with strongly “inelastic” events for a wide range of the essential parameters of the problem.

We have observed and visualized the process of the $SU(2)$ symmetry restoration in some finite space-time region as a result of the collisions of the intense gauge pulses. At last but not least, it is important to emphasize the observed correlation between the occurrence of the inelastic events (for non-parallel isospin configurations) and the restoration of the symmetry in high energy collisions. Both these phenomena require the same order of the amplitude of the initial configurations.

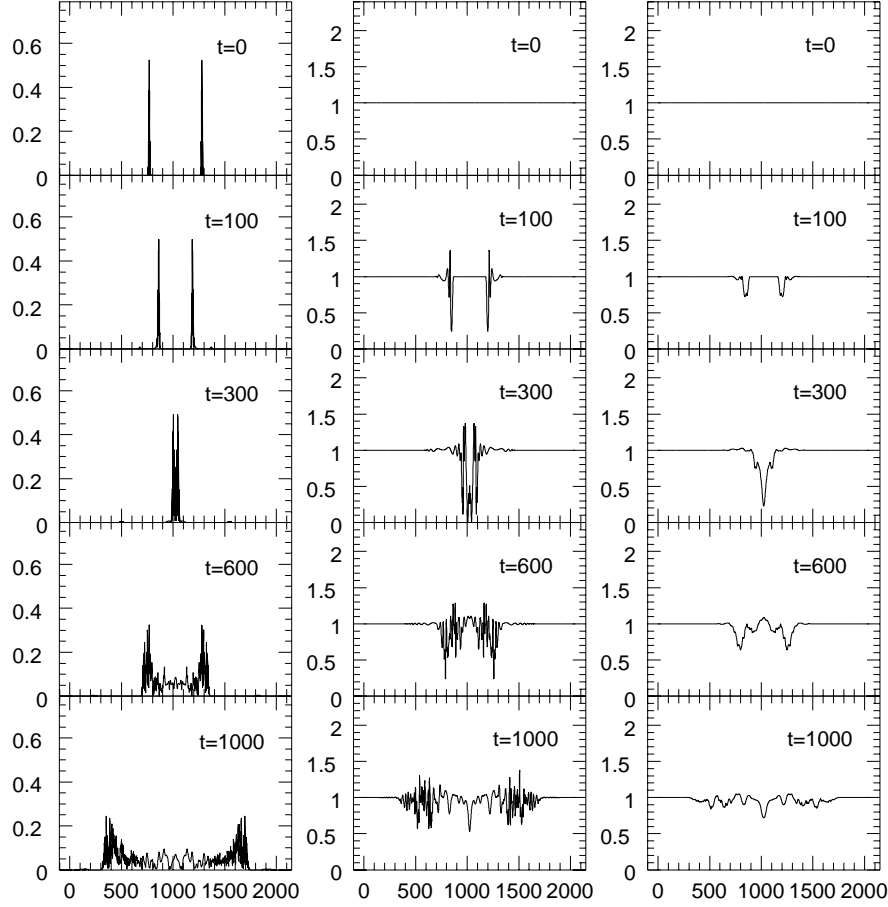


Figure 3: Space-time development of symmetry restoration induced by two colliding gauge wave packets with orthogonal isospins in the presence of the Higgs vacuum condensate. First column shows the scaled gauge field $|A|/v$ as a function of space coordinate z five chosen times. Second column demonstrates the corresponding space-time evolution of the scaled Higgs field $|\Phi|/v$. Third column shows the scaled Higgs field after smoothing over 50 lattice sites. This simulation was done on a lattice of sites $n = 2048$ and lattice spacing $a = 1$. The parameters were $\bar{k} = \pi/4$, $\Delta k = \pi/16$, $M_H = M_W = 0.15$, $g = 0.65$, and $\sigma = 1$.

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